Joint remote preparation of a four-dimensional quantum state

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Abstract

We propose various protocols for joint remotely prepare a four-dimensional quantum state by using two- and three-particle four-dimensional entangled state as the quantum channel. The single- and two-particle generalized projective measurement and the appropriate unitary operation are needed in our protocols. It is shown that the receiver can reconstruct the unknown original state only if two senders collaborate with each other.

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1. Introduction

Quantum entanglement plays a more and more critical role in quantum information theory. Quantum teleportation, proposed by Bennett *et al*[1], is the process that transmits an unknown quantum state from a sender to spatially distant receiver using the entanglement channel with the help of some classical information. In the last decade, Lo[2], Pati[3], and Bennett *et al*[4] presented a new quantum communication protocol that uses classical communication and a previously shared entangled resource to remotely prepare a quantum state. This communication protocol is called remote state preparation(RSP). RSP is another important protocol taking advantage of entanglement, in which the sender Alice performs a measurement on her share of the entangled resource in a basis chosen in accordance with the state she wishes to help the receiver Bob in his laboratory to prepare. In RSP, Alice is assumed to know fully the transmitted state to be prepared by Bob, so RSP is called the teleportation of a known state. Compared with teleportation. RSP requires less classical communication cost than teleportation[3]. In recent years, RSP has attracted much attention, various protocols[5-12] for generalization of RSP have been presented using various kinds of methods, including low-entanglement RSP[5], optimal RSP[6], generalized RSP[7], oblivious RSP[8], continuous variable RSP[9,10], etc. Several RSP protocols in higher dimensional Hilbert space have been proposed[13-15]. Meanwhile, some RSP protocols have already been implemented experimentally[16-20].

All the above protocols assume the case that only one sender knows the original state. However, if two-party or multiparty share an original quantum state, and they want to remotely prepare it in the laboratory of receiver, how can they do it ? To answer this problem, recently, a novel aspect of RSP, called as the joint

remote state preparation(JRSP), has been proposed[21-25]. In these protocols of the JRSP[21-25], two senders(or N senders) know partly of the original state they want to remotely prepare, respectively. If and only if all the senders agree to collaborate, the receiver can reconstruct the original quantum state. Nevertheless, in Refs.[21-25] only the single- or multi-qubit state was considered. Though various protocols of RSP of high-dimensional quantum state have been proposed in recent years[13-15], but no scheme has yet been reported for the JRSP of higher-dimensional quantum state.

During the last few years, the high-dimensional system in quantum information processing(QIP) has attracted much attention. High-dimensional systems have properties which are different from qubit counterparts which could be useful for QIP. For instance, we can note that high-dimensional systems can be more entangled than qubits[26-28] and can share a larger fraction of their entanglement[29]. These properties, as well as the larger dimension alone could aid many QIP tasks, including quantum key distribution[30-33], quantum teleportation[1,34], quantum bit commitment[35,36], quantum computing[37-41], quantum dense coding[42], quantum secure communication[43], quantum secret sharing[44,45], and quantum state remotely preparation[13-15]. In this paper, we propose a set of protocols for two senders to remotely prepare the single- and two-particle four-dimensional(FD) quantum state by using various types of quantum channel. In our protocols, the single- and two-particle FD projective measurement and appropriate unitary operation are needed. This paper is organized as follows. In section 2, a protocol for joint remotely prepare an unknown single-particle FD quantum state by using a tripartite FD entangled state as quantum channel is presented. In section 3, we propose two protocols of joint remote preparation of an

3

unknown bipartite FD entangled state via two tripartite and three bipartite FD entangled states as quantum channel, respectively. Conclusions are given in section 4.

2. Joint remote preparation of a single-particle FD quantum state

Suppose that Alice$_1$, and Alice$_2$ share the original state $|\varphi\rangle$, and they wish to help Bob remotely prepare a FD quantum state

$$|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle + \delta|3\rangle, \tag{1}$$

like with Ref.[13-15], here we only consider that α, β, γ and δ are real and $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$. Suppose that Alice$_1$ and Alice$_2$ know the original state $|\varphi\rangle$ partly, that is, Alice$_1$ knows α_1, β_1, γ_1 and δ_1, Alice$_2$ knows α_2, β_2, γ_2 and δ_2, where $\alpha_1\alpha_2 = \alpha$, $\beta_1\beta_2 = \beta$, $\gamma_1\gamma_2 = \gamma$ and $\delta_1\delta_2 = \delta$. We also assume that the quantum channel shared by Alice$_1$, Alice$_2$ and Bob is the tripartite FD entangled state

$$|\phi\rangle_{123} = \frac{1}{2}\sum_{j=0}^{3}|jjj\rangle_{123}. \tag{2}$$

Assume particle 1 belongs to Alice$_1$, particle 2 to Alice$_2$, and particle 3 to Bob, respectively. In order to help Bob remotely prepare the original state, what Alice$_1$ and Alice$_2$ need to do is to perform single-particle FD projective measurements on their own particles 1 and 2 respectively. The measurement basis chosen by Alice$_1$ and Alice$_2$ are the set of mutually orthogonal basis vectors (MOBVs) $\{|\psi_0^{(k)}\rangle\}, \{|\psi_1^{(k)}\rangle\}, \{|\psi_2^{(k)}\rangle\}, \{|\psi_3^{(k)}\rangle\}$ which is given by

$$|\psi_0^{(k)}\rangle = \alpha_k|0\rangle + \beta_k|1\rangle + \gamma_k|2\rangle + \delta_k|3\rangle,$$
$$|\psi_1^{(k)}\rangle = \beta_k|0\rangle - \alpha_k|1\rangle + \delta_k|2\rangle - \gamma_k|3\rangle,$$

4

$$|\psi_2^{(k)}\rangle = -\gamma_k|0\rangle + \delta_k|1\rangle + \alpha_k|2\rangle - \beta_k|3\rangle,$$

$$|\psi_3^{(k)}\rangle = -\delta_k|0\rangle - \gamma_k|1\rangle + \beta_k|2\rangle + \alpha_k|3\rangle, \tag{3}$$

where $k = 1$ and 2, and $\{|\psi_j^{(1)}\rangle\}$ ($j = 0, 1, 2, 3$) is a set of MOBVs chosen by Alice$_1$, and $\{|\psi_j^{(2)}\rangle\}$ by Alice$_2$. Since $\alpha_1, \beta_1, \gamma_1$ and δ_1 ($\alpha_2, \beta_2, \gamma_2$ and δ_2) that are necessary for determining the basis $\{|\psi_j^{(1)}\rangle\}$ ($\{|\psi_j^{(2)}\rangle\}$) are known only to Alice$_1$ (Alice$_2$), so Alice$_1$ and Alice$_2$ are always able to make the measurements independently of each other.

By Eq.(3), the state (2) can be described as

$$\begin{aligned}
|\phi\rangle_{123} = \frac{1}{2}[&|\psi_0^{(1)}\rangle_1|\psi_0^{(2)}\rangle_2(\alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle + \delta|3\rangle)_3 \\
+&|\psi_1^{(1)}\rangle_1|\psi_1^{(2)}\rangle_2(\beta|0\rangle + \alpha|1\rangle + \delta|2\rangle + \gamma|3\rangle)_3 \\
+&|\psi_2^{(1)}\rangle_1|\psi_2^{(2)}\rangle_2(\gamma|0\rangle + \delta|1\rangle + \alpha|2\rangle + \beta|3\rangle)_3 \\
+&|\psi_3^{(1)}\rangle_1|\psi_3^{(2)}\rangle_2(\delta|0\rangle + \gamma|1\rangle + \beta|2\rangle + \alpha|3\rangle)_3 \\
+&|\psi_0^{(1)}\rangle_1|\psi_1^{(2)}\rangle_2(\alpha_1\beta_1|0\rangle - \beta_1\alpha_2|1\rangle + \gamma_1\delta_2|2\rangle - \delta_1\gamma_2|3\rangle)_3 + \cdots],
\end{aligned} \tag{4}$$

where "$\cdots$" represents 11 terms with $l \neq m$ in $|\psi_l^{(1)}\rangle_1|\psi_m^{(2)}\rangle_2$ ($l, m = 0, 1, 2, 3$). Clearly, in Eq.(4) only the four first terms can cause success, but all the 12 remaining terms with $l \neq m$ lead to failure. Now let Alice$_1$ and Alice$_2$ perform single-particle FD projective measurements on their own particles 1 and 2 respectively, and then they inform Bob of their results by the classical channels. According to the measurement results of Alice$_1$ and Alice$_2$, the receiver Bob can reconstruct the original state at his side. Without loss of generality, assume Alice$_1$'s measurement result is $|\psi_1^{(1)}\rangle_1$ and Alice$_2$'s result is $|\psi_1^{(2)}\rangle_2$, the particle 3 will collapse into the state $\frac{1}{2}(\beta|0\rangle + \alpha|1\rangle + \delta|2\rangle + \gamma|3\rangle)_3$. Bob needs to perform a local unitary operation U_1 on particle 4, the state of particle 3 will evolve $\frac{1}{3}(\alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle + \delta|3\rangle)_3$,

5

Table 1: Corresponding relation between the measurement results(MR) of Alice$_1$ and Alice$_2$ and unitary operation U_i by Bob

MR	U_i
$|\psi_0^{(1)}\rangle_1|\psi_0^{(2)}\rangle_2$	U_0
$|\psi_1^{(1)}\rangle_1|\psi_1^{(2)}\rangle_2$	U_1
$|\psi_2^{(1)}\rangle_1|\psi_2^{(2)}\rangle_2$	U_2
$|\psi_3^{(1)}\rangle_1|\psi_3^{(2)}\rangle_2$	U_3

which is exactly the original state $|\varphi\rangle$. Here the unitary operation U_1 is one of the unitary operations $U_i(i = 0, 1, 2, 3)$

$$U_0 = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}, U_1 = \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_x \end{pmatrix}, U_2 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, U_3 = \begin{pmatrix} 0 & \sigma_x \\ \sigma_x & 0 \end{pmatrix}, \quad (5)$$

where I is 2×2 identity matrix and σ_x is Pauli matrix. If the measurement results of Alice$_1$ and Alice$_2$ are the other 3 cases of the four first terms in Eq.(4), the relation between the results obtained by Alice$_1$ and Alice$_2$ and the unitary operations performed by Bob is shown in Table 1. The required classical communication cost is 4 bits ($2 \times \log_2 4$) in the protocol.

3. Joint remote preparation of a bipartite FD entangled state

We now consider the situation when the state of joint remote preparation is a bipartite FD entangled state. In what follows we present two protocols of JRSP using different quantum resources as the quantum channel. The first protocol relies two tripartite FD entangled states and the second protocol uses three bipartite FD entangled states as the quantum channel, respectively.

6

3.1. JRSP by using two tripartite FD entangled states as the quantum channel

Suppose that $Alice_1$ and $Alice_2$ wish to help the receiver Bob remotely prepare a bipartite FD entangled state

$$|\psi\rangle = a|00\rangle + b|11\rangle + c|22\rangle + d|33\rangle, \tag{6}$$

where a, b, c and d are real and $a^2 + b^2 + c^2 + d^2 = 1$. Assume that $Alice_1$ and $Alice_2$ know the original state $|\psi\rangle$ partly, i.e., $Alice_1$ knows a_1, b_1, c_1 and d_1, $Alice_2$ knows a_2, b_2, c_2 and d_2, where $a_1 a_2 = a, b_1 b_2 = b, c_1 c_2 = c$ and $d_1 d_2 = d$. We also suppose that the quantum channels shared by $Alice_1$, $Alice_2$ and Bob are two tripartite FD entangled states

$$|\phi\rangle_{123} = \frac{1}{2} \sum_{j=0}^{3} |jjj\rangle_{123},$$

$$|\phi\rangle_{456} = \frac{1}{2} \sum_{j=0}^{3} |jjj\rangle_{456}. \tag{7}$$

Here, particles 1 and 4 belong to $Alice_1$, particles 2 and 5 to $Alice_2$ and particle 3 and 6 to Bob, respectively. In order to help Bob to remotely prepare the original state, $Alice_1$ and $Alice_2$ should perform the two-particle FD projective measurements on their own particles $(1, 4)$ and $(2, 5)$, respectively. The measurement basis chosen by $Alice_1$ and $Alice_2$ are the set of MOBVs $\{|\psi_{gh}^{(k)}\rangle\}$ $(g, h = 0, 1, 2, 3, k = 1, 2)$

$$|\psi_{00}^{(k)}\rangle = a_k|00\rangle + b_k|11\rangle + c_k|22\rangle + d_k|33\rangle,$$

$$|\psi_{10}^{(k)}\rangle = b_k|00\rangle - a_k|11\rangle + d_k|22\rangle - c_k|33\rangle,$$

$$|\psi_{20}^{(k)}\rangle = -c_k|00\rangle + d_k|11\rangle + a_k|22\rangle - b_k|33\rangle,$$

$$|\psi_{30}^{(k)}\rangle = -d_k|00\rangle - c_k|11\rangle + b_k|22\rangle + a_k|33\rangle,$$

7

$$|\psi_{01}^{(k)}\rangle = a_k|01\rangle + b_k|12\rangle + c_k|23\rangle + d_k|30\rangle,$$

$$|\psi_{11}^{(k)}\rangle = b_k|01\rangle - a_k|12\rangle + d_k|23\rangle - c_k|30\rangle,$$

$$|\psi_{21}^{(k)}\rangle = -c_k|01\rangle + d_k|12\rangle + a_k|23\rangle - b_k|30\rangle,$$

$$|\psi_{31}^{(k)}\rangle = -d_k|01\rangle - c_k|12\rangle + b_k|23\rangle + a_k|30\rangle,$$

$$|\psi_{02}^{(k)}\rangle = a_k|02\rangle + b_k|13\rangle + c_k|20\rangle + d_k|31\rangle,$$

$$|\psi_{12}^{(k)}\rangle = b_k|02\rangle - a_k|13\rangle + d_k|20\rangle - c_k|31\rangle,$$

$$|\psi_{22}^{(k)}\rangle = -c_k|02\rangle + d_k|13\rangle + a_k|20\rangle - b_k|31\rangle,$$

$$|\psi_{32}^{(k)}\rangle = -d_k|02\rangle - c_k|13\rangle + b_k|20\rangle + a_k|31\rangle,$$

$$|\psi_{03}^{(k)}\rangle = a_k|03\rangle + b_k|10\rangle + c_k|21\rangle + d_k|32\rangle,$$

$$|\psi_{13}^{(k)}\rangle = b_k|03\rangle - a_k|10\rangle + d_k|21\rangle - c_k|32\rangle,$$

$$|\psi_{23}^{(k)}\rangle = -c_k|03\rangle + d_k|10\rangle + a_k|21\rangle - b_k|32\rangle,$$

$$|\psi_{33}^{(k)}\rangle = -d_k|03\rangle - c_k|10\rangle + b_k|21\rangle + a_k|32\rangle, \tag{8}$$

where $\{|\psi_{gh}^{(1)}\rangle\}$ is a set of MOBVs chosen by Alice$_1$, and $\{|\psi_{gh}^{(2)}\rangle\}$ by Alice$_2$. From Eq.(8), the quantum channel composed of entangled states (7) can be written as

$$
\begin{aligned}
|\Phi\rangle &= |\phi\rangle_{123} \otimes |\phi\rangle_{456} \\
&= \frac{1}{4}[|\psi_{00}^{(1)}\rangle_{14}|\psi_{00}^{(2)}\rangle_{25}(a|00\rangle + b|11\rangle + c|22\rangle + d|33\rangle)_{36} \\
&\quad + |\psi_{10}^{(1)}\rangle_{14}|\psi_{10}^{(2)}\rangle_{25}(b|00\rangle + a|11\rangle + d|22\rangle + c|33\rangle)_{36} \\
&\quad + |\psi_{20}^{(1)}\rangle_{14}|\psi_{20}^{(2)}\rangle_{25}(c|00\rangle + d|11\rangle + a|22\rangle + b|33\rangle)_{36} \\
&\quad + |\psi_{30}^{(1)}\rangle_{14}|\psi_{30}^{(2)}\rangle_{25}(d|00\rangle + c|11\rangle + b|22\rangle + a|33\rangle)_{36} \\
&\quad + |\psi_{01}^{(1)}\rangle_{14}|\psi_{01}^{(2)}\rangle_{25}(a|01\rangle + b|12\rangle + c|23\rangle + d|30\rangle)_{36} \\
&\quad + |\psi_{11}^{(1)}\rangle_{14}|\psi_{11}^{(2)}\rangle_{25}(b|01\rangle + a|12\rangle + d|23\rangle + c|30\rangle)_{36} \\
&\quad + |\psi_{21}^{(1)}\rangle_{14}|\psi_{21}^{(2)}\rangle_{25}(c|01\rangle + d|12\rangle + a|23\rangle + b|30\rangle)_{36}
\end{aligned}
$$

8

$$+|\psi_{31}^{(1)}\rangle_{14}|\psi_{31}^{(2)}\rangle_{25}(d|01\rangle + c|12\rangle + b|23\rangle + a|30\rangle)_{36}$$

$$+|\psi_{02}^{(1)}\rangle_{14}|\psi_{02}^{(2)}\rangle_{25}(a|02\rangle + b|13\rangle + c|20\rangle + d|31\rangle)_{36}$$

$$+|\psi_{12}^{(1)}\rangle_{14}|\psi_{12}^{(2)}\rangle_{25}(b|02\rangle + a|13\rangle + d|20\rangle + c|31\rangle)_{36}$$

$$+|\psi_{22}^{(1)}\rangle_{14}|\psi_{22}^{(2)}\rangle_{25}(c|02\rangle + d|13\rangle + a|20\rangle + b|31\rangle)_{36}$$

$$+|\psi_{32}^{(1)}\rangle_{14}|\psi_{32}^{(2)}\rangle_{25}(d|02\rangle + c|13\rangle + b|20\rangle + a|31\rangle)_{36}$$

$$+|\psi_{03}^{(1)}\rangle_{14}|\psi_{03}^{(2)}\rangle_{25}(a|03\rangle + b|10\rangle + c|21\rangle + d|32\rangle)_{36}$$

$$+|\psi_{13}^{(1)}\rangle_{14}|\psi_{13}^{(2)}\rangle_{25}(b|03\rangle + a|10\rangle + d|21\rangle + c|32\rangle)_{36}$$

$$+|\psi_{23}^{(1)}\rangle_{14}|\psi_{23}^{(2)}\rangle_{25}(c|03\rangle + d|10\rangle + a|21\rangle + b|32\rangle)_{36}$$

$$+|\psi_{33}^{(1)}\rangle_{14}|\psi_{33}^{(2)}\rangle_{25}(d|03\rangle + c|10\rangle + b|21\rangle + a|32\rangle)_{36}$$

$$+|\psi_{00}^{(1)}\rangle_{14}|\psi_{10}^{(2)}\rangle_{25}(a_1b_2|00\rangle - a_2b_1|11\rangle + c_1d_2|22\rangle - c_2d_1|33\rangle)_{36} + \cdots], \qquad (9)$$

where "$\cdots$" includes 47 other terms with $g \neq m$ or/and $h \neq n$ in $|\psi_{gh}^{(1)}\rangle_{14}|\psi_{mn}^{(2)}\rangle_{25}$ ($g, h, m, n = 0, 1, 2, 3$). In Eq.(8) only the 16 first terms give rise to a success, all the 48 remaining terms with $g \neq m$ or/and $h \neq n$ lead to a failure. Now let Alice$_1$ and Alice$_2$ perform the two-particle FD projective measurements on their own particles $(1, 4)$ and $(2, 5)$, respectively, and then they inform Bob of their outcomes in public. In accord with the measurement outcomes of Alice$_1$ and Alice$_2$, Bob can reconstruct the original state. For instance, suppose Alice$_1$'s measurement outcome is $|\psi_{11}^{(1)}\rangle_{14}$ and Alice$_2$'s outcome is $|\psi_{11}^{2}\rangle_{25}$, the particles 3 and 6 will collapse into the state $\frac{1}{4}(b|01\rangle + a|12\rangle + d|23\rangle + c|30\rangle)_{36}$. According to Alice$_1$'s and Alice$_2$'s public announcement, Bob should perform the unitary operations $U_1 \otimes U_5$ on particles 3 and 6, thus the bipartite FD entangled state (6) can be reconstructed. Here unitary operation U_1 is defined by Eq.(5) and U_5 is

Table 2: Corresponding relation between the measurement results (MR) of Alice$_1$ and Alice$_2$ and the local unitary operations $(U_i)_3 \otimes (U_j)_6$ $(i, j = 0, 1, \cdots, 7)$ performed by Bob. $(\zeta_{gh} \rightarrow |\psi_{gh}^{(1)}\rangle_{14}$, $\eta_{mn} \rightarrow |\psi_{mn}^{(2)}\rangle_{25}$, $u_i \rightarrow (U_i)_3$, $v_j \rightarrow (U_j)_6$, $g, h, m, n = 0, 1, 2, 3)$

MR	$u_i \otimes v_j$	MR	$u_i \otimes v_j$
$\zeta_{00}\eta_{00}$	$u_0 \otimes v_0$	$\zeta_{02}\eta_{02}$	$u_0 \otimes v_2$
$\zeta_{10}\eta_{10}$	$u_1 \otimes v_1$	$\zeta_{12}\eta_{12}$	$u_1 \otimes v_3$
$\zeta_{20}\eta_{20}$	$u_2 \otimes v_2$	$\zeta_{22}\eta_{22}$	$u_2 \otimes v_0$
$\zeta_{30}\eta_{30}$	$u_3 \otimes v_3$	$\zeta_{32}\eta_{32}$	$u_3 \otimes v_1$
$\zeta_{01}\eta_{01}$	$u_1 \otimes v_4$	$\zeta_{03}\eta_{03}$	$u_0 \otimes v_6$
$\zeta_{11}\eta_{11}$	$u_1 \otimes v_5$	$\zeta_{13}\eta_{13}$	$u_1 \otimes v_7$
$\zeta_{21}\eta_{21}$	$u_2 \otimes v_6$	$\zeta_{23}\eta_{23}$	$u_2 \otimes v_4$
$\zeta_{31}\eta_{31}$	$u_3 \otimes v_7$	$\zeta_{33}\eta_{33}$	$u_3 \otimes v_5$

one of the unitary operations U_j $(j = 4, 5, 6, 7)$

$$U_4 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \quad U_5 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$U_6 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad U_7 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{10}$$

If the measurement outcomes of Alice$_1$ and Alice$_2$ are the other 15 cases of the sixteen first terms in Eq.(9), the relation between the outcomes by Alice$_1$ and

Alice$_2$ and the unitary operations by Bob is shown in Table 2. The required classical communication cost is 8 bits in this protocol.

3.2. JRSP by using three bipartite FD entangled states as the quantum channel

Suppose the state that Alice$_1$ and Alice$_2$ wish to help Bob remotely prepare is still in state $|\psi\rangle$ (see Eq.(6)). We also assume that Alice$_1$, Alice$_2$ and Bob share three bipartite FD entangled states as quantum channel

$$|\phi\rangle_{12} = \frac{1}{2} \sum_{j=0}^{3} |jj\rangle_{12}$$

$$|\phi\rangle_{34} = \frac{1}{2} \sum_{j=0}^{3} |jj\rangle_{34}$$

$$|\phi\rangle_{56} = \frac{1}{2} \sum_{j=0}^{3} |jj\rangle_{56}, \tag{11}$$

where particles 1 and 3 belong to Alice$_1$, particles 2 and 5 to Alice$_2$ and particles 4 and 6 to Bob, respectively. As in the previous protocol, Alice$_1$ and Alice$_2$ perform the two-particle FD projective measurements on their own particles $(1,3)$ and $(2,5)$, respectively. The measurement basis chosen by Alice$_1$ and Alice$_2$ is still in $\{|\psi_{gh}^k\rangle\}$ (see Eq.(8)). The quantum channel $|\Phi\rangle = |\phi\rangle_{12}|\phi\rangle_{34}|\phi\rangle_{56}$ can be written in terms of basis $\{|\psi_{gh}^k\rangle\}$ as

$$
\begin{aligned}
|\Psi\rangle = \frac{1}{8} \sum_{j=0}^{3} [&|G_{0j}\rangle(a|\lambda_{0j}\rangle + b|\lambda_{1j}\rangle + c|\lambda_{2j}\rangle + d|\lambda_{3j}\rangle)_{46} \\
+&|G_{1j}\rangle(b|\lambda_{0j}\rangle + a|\lambda_{1j}\rangle + d|\lambda_{2j}\rangle + c|\lambda_{3j}\rangle)_{46} \\
+&|G_{2j}\rangle(c|\lambda_{0j}\rangle + d|\lambda_{1j}\rangle + a|\lambda_{2j}\rangle + b|\lambda_{3j}\rangle)_{46} \\
+&|G_{3j}\rangle(d|\lambda_{0j}\rangle + c|\lambda_{2j}\rangle + b|\lambda_{2j}\rangle + a|\lambda_{3j}\rangle)_{46} \\
+&|G_{4j}\rangle(a|\lambda_{1j}\rangle + b|\lambda_{2j}\rangle + c|\lambda_{3j}\rangle + d|\lambda_{0j}\rangle)_{46}
\end{aligned}
$$

11

$$+|G_{5j}\rangle(b|\lambda_{1j}\rangle + a|\lambda_{2j}\rangle + d|\lambda_{3j}\rangle + c|\lambda_{0j}\rangle)_{46}$$

$$+|G_{6j}\rangle(c|\lambda_{1j}\rangle + d|\lambda_{2j}\rangle + a|\lambda_{3j}\rangle + b|\lambda_{0j}\rangle)_{46}$$

$$+|G_{7j}\rangle(d|\lambda_{1j}\rangle + c|\lambda_{2j}\rangle + b|\lambda_{3j}\rangle + a|\lambda_{0j}\rangle)_{46}$$

$$+|\psi_{00}^{(1)}\rangle_{13}|\psi_{10}^{(2)}\rangle_{25}(a_1b_2|00\rangle - b_1a_2|11\rangle + c_1d_2|22\rangle - d_1c_2|33\rangle)_{46}$$

$$+\cdots], \qquad (12)$$

where $|G_{pj}\rangle \equiv |G_{pj}\rangle_{1325}$ given in appendix A and $|G_{0j}\rangle \sim |G_{7j}\rangle$ include 64 terms with $g = m$ or/and $h \neq n$ in $|\psi_{gh}^{(1)}\rangle_{13}|\psi_{mn}^{(2)}\rangle_{25}$ ($g, h, m, n = 0, 1, 2, 3$), "$\cdots$" includes 191 other terms with $g \neq m$ or/and $h \neq n$ in $|\psi_{gh}^{(1)}\rangle_{13}|\psi_{mn}^{(2)}\rangle_{25}$, $|\lambda_{ij}\rangle \equiv |i, i \oplus j\rangle$ and $i \oplus j$ means $i + j$ mod 3. In Eq.(12) only the 64 first terms (i.e. $|G_{0j}\rangle \sim |G_{7j}\rangle$) can cause success, but all the 192 remaining terms with $g \neq m$ lead to failure. For example, assume Alice$_1$'s measurement result is $|\psi_{20}^{(1)}\rangle_{13}$ and Alice$_2$'s result is $|\psi_{21}^{2}\rangle_{25}$, the particles 4 and 6 will collapse into the state $\frac{1}{8}(c|01\rangle + d|12\rangle + a|23\rangle + b|30\rangle)_{46}$, then Bob should perform $U_2 \otimes U_6$ on particles 4 and 6, the original state(6) can be reconstructed successfully. If the measurement results of Alice$_1$ and Alice$_2$ are the other 63 cases of the successful terms in Eq.(12), the relation between the results by Alice$_1$ and Alice$_2$ and the unitary operations by Bob is shown in Table 3. In this protocol, the required classical communication cost is also 8 bits.

4. Conclusion

We propose the protocols for joint remote preparation of the four-dimensional quantum states by using various types of the four-dimensional entangled states as quantum channel. In these protocols, two senders share an original state which they wish to help the receiver to remotely prepare it, but each sender only partly knows the state. It is shown that, only if when all the senders collaborate with

Table 3: Corresponding relation between the measurement results (MR) of Alice$_1$ and Alice$_2$ and the local unitary operations $(U_i)_4 \otimes (U_j)_6$ $(i, j = 0, 1, \cdots, 7)$ by Bob. ($\xi_{gh} \to |\psi_{gh}^{(1)}\rangle_{13}$, $\tau_{mn} \to |\psi_{mn}^{(2)}\rangle_{25}$, $u_i \to (U_i)_4$, $v_j \to (U_j)_6$, $g, h, m, n = 0, 1, 2, 3$).

MR	$u_i \otimes v_j$	MR	$u_i \otimes v_j$
$\xi_{00}\tau_{00}(or\ \xi_{22}\tau_{22})$	$u_0 \otimes v_0$	$\xi_{01}\tau_{00}(or\ \xi_{23}\tau_{22})$	$u_4 \otimes v_0$
$\xi_{00}\tau_{01}(or\ \xi_{22}\tau_{23})$	$u_0 \otimes v_4$	$\xi_{01}\tau_{01}(or\ \xi_{23}\tau_{23})$	$u_4 \otimes v_4$
$\xi_{00}\tau_{02}(or\ \xi_{22}\tau_{20})$	$u_0 \otimes v_2$	$\xi_{01}\tau_{02}(or\ \xi_{23}\tau_{20})$	$u_4 \otimes v_2$
$\xi_{00}\tau_{03}(or\ \xi_{22}\tau_{21})$	$u_0 \otimes v_6$	$\xi_{01}\tau_{03}(or\ \xi_{23}\tau_{21})$	$u_4 \otimes v_6$
$\xi_{10}\tau_{10}(or\ \xi_{32}\tau_{32})$	$u_1 \otimes v_1$	$\xi_{11}\tau_{10}(or\ \xi_{33}\tau_{32})$	$u_5 \otimes v_1$
$\xi_{10}\tau_{11}(or\ \xi_{32}\tau_{33})$	$u_1 \otimes v_5$	$\xi_{11}\tau_{11}(or\ \xi_{33}\tau_{33})$	$u_5 \otimes v_5$
$\xi_{10}\tau_{12}(or\ \xi_{32}\tau_{30})$	$u_1 \otimes v_3$	$\xi_{11}\tau_{12}(or\ \xi_{33}\tau_{30})$	$u_5 \otimes v_3$
$\xi_{10}\tau_{13}(or\ \xi_{32}\tau_{31})$	$u_1 \otimes v_7$	$\xi_{11}\tau_{13}(or\ \xi_{33}\tau_{31})$	$u_5 \otimes v_7$
$\xi_{20}\tau_{20}(or\ \xi_{02}\tau_{02})$	$u_2 \otimes v_2$	$\xi_{21}\tau_{20}(or\ \xi_{03}\tau_{02})$	$u_6 \otimes v_2$
$\xi_{20}\tau_{21}(or\ \xi_{02}\tau_{03})$	$u_2 \otimes v_6$	$\xi_{21}\tau_{21}(or\ \xi_{03}\tau_{03})$	$u_6 \otimes v_6$
$\xi_{20}\tau_{22}(or\ \xi_{02}\tau_{00})$	$u_2 \otimes v_0$	$\xi_{21}\tau_{22}(or\ \xi_{03}\tau_{00})$	$u_6 \otimes v_0$
$\xi_{20}\tau_{23}(or\ \xi_{02}\tau_{01})$	$u_2 \otimes v_4$	$\xi_{21}\tau_{23}(or\ \xi_{03}\tau_{01})$	$u_6 \otimes v_4$
$\xi_{30}\tau_{30}(or\ \xi_{12}\tau_{12})$	$u_3 \otimes v_3$	$\xi_{31}\tau_{30}(or\ \xi_{13}\tau_{12})$	$u_7 \otimes v_3$
$\xi_{30}\tau_{31}(or\ \xi_{12}\tau_{13})$	$u_3 \otimes v_7$	$\xi_{31}\tau_{31}(or\ \xi_{13}\tau_{13})$	$u_7 \otimes v_7$
$\xi_{30}\tau_{32}(or\ \xi_{12}\tau_{10})$	$u_3 \otimes v_1$	$\xi_{31}\tau_{32}(or\ \xi_{13}\tau_{10})$	$u_7 \otimes v_1$
$\xi_{30}\tau_{33}(or\ \xi_{12}\tau_{11})$	$u_3 \otimes v_5$	$\xi_{31}\tau_{33}(or\ \xi_{13}\tau_{11})$	$u_7 \otimes v_5$

each other, the receiver can remotely reconstruct the original state. In order to realize the JRSP, two senders need to perform four-dimensional projective measurements on their own particle, respectively, and then inform the receiver Bob of the measurement outcomes through the classical channel. According to the public information of the senders, the receiver can obtain the original state by using some appropriate unitary operations. These protocols require resources such as bipartite or tripartite four-dimensional entangled state as the quantum channel, single- or two-particle four-dimensional projective measurement, classical communication and appropriate unitary operation. In principle, our protocols can be generalized to the case of JRSP of d-dimensional ($d = 2^N$, N is a positive integer greater than 2) quantum state. Furthermore, the required classical communication cost in the JRSP process in our protocols has been calculated respectively.

Appendix A.

The states $|G_{pj}\rangle$ ($p = 0, 1, \cdots, 7$; $j = 0, 1, 2, 3$) in Eq.(12) are of the form

$$|G_{00}\rangle = |\psi_{00}^1\rangle_{13}|\psi_{00}^2\rangle_{25} + |\psi_{22}^1\rangle_{13}|\psi_{22}^2\rangle_{25}, \tag{A. 1}$$

$$|G_{01}\rangle = |\psi_{00}^1\rangle_{13}|\psi_{01}^2\rangle_{25} + |\psi_{22}^1\rangle_{13}|\psi_{23}^2\rangle_{25}, \tag{A. 2}$$

$$|G_{02}\rangle = |\psi_{00}^1\rangle_{13}|\psi_{02}^2\rangle_{25} + |\psi_{22}^1\rangle_{13}|\psi_{20}^2\rangle_{25}, \tag{A. 3}$$

$$|G_{03}\rangle = |\psi_{00}^1\rangle_{13}|\psi_{03}^2\rangle_{25} + |\psi_{22}^1\rangle_{13}|\psi_{21}^2\rangle_{25}, \tag{A. 4}$$

$$|G_{10}\rangle = |\psi_{10}^1\rangle_{13}|\psi_{10}^2\rangle_{25} + |\psi_{32}^1\rangle_{13}|\psi_{32}^2\rangle_{25}, \tag{A. 5}$$

$$|G_{11}\rangle = |\psi_{10}^1\rangle_{13}|\psi_{11}^2\rangle_{25} + |\psi_{32}^1\rangle_{13}|\psi_{33}^2\rangle_{25}, \tag{A. 6}$$

$$|G_{12}\rangle = |\psi_{10}^1\rangle_{13}|\psi_{12}^2\rangle_{25} + |\psi_{32}^1\rangle_{13}|\psi_{30}^2\rangle_{25}, \tag{A. 7}$$

$$|G_{13}\rangle = |\psi_{10}^1\rangle_{13}|\psi_{13}^2\rangle_{25} + |\psi_{32}^1\rangle_{13}|\psi_{31}^2\rangle_{25}, \tag{A. 8}$$

$$|G_{20}\rangle = |\psi_{20}^1\rangle_{13}|\psi_{20}^2\rangle_{25} + |\psi_{02}^1\rangle_{13}|\psi_{02}^2\rangle_{25}, \tag{A. 9}$$

$$|G_{21}\rangle = |\psi_{20}^1\rangle_{13}|\psi_{21}^2\rangle_{25} + |\psi_{02}^1\rangle_{13}|\psi_{03}^2\rangle_{25}, \tag{A. 10}$$

$$|G_{22}\rangle = |\psi_{20}^1\rangle_{13}|\psi_{22}^2\rangle_{25} + |\psi_{02}^1\rangle_{13}|\psi_{00}^2\rangle_{25}, \tag{A. 11}$$

$$|G_{23}\rangle = |\psi_{20}^1\rangle_{13}|\psi_{23}^2\rangle_{25} + |\psi_{02}^1\rangle_{13}|\psi_{01}^2\rangle_{25}, \tag{A. 12}$$

$$|G_{30}\rangle = |\psi_{30}^1\rangle_{13}|\psi_{30}^2\rangle_{25} + |\psi_{12}^1\rangle_{13}|\psi_{12}^2\rangle_{25}, \tag{A. 13}$$

$$|G_{31}\rangle = |\psi_{30}^1\rangle_{13}|\psi_{31}^2\rangle_{25} + |\psi_{12}^1\rangle_{13}|\psi_{13}^2\rangle_{25}, \tag{A. 14}$$

$$|G_{32}\rangle = |\psi_{30}^1\rangle_{13}|\psi_{32}^2\rangle_{25} + |\psi_{12}^1\rangle_{13}|\psi_{10}^2\rangle_{25}, \tag{A. 15}$$

$$|G_{33}\rangle = |\psi_{30}^1\rangle_{13}|\psi_{33}^2\rangle_{25} + |\psi_{12}^1\rangle_{13}|\psi_{11}^2\rangle_{25}, \tag{A. 16}$$

$$|G_{40}\rangle = |\psi_{01}^1\rangle_{13}|\psi_{00}^2\rangle_{25} + |\psi_{23}^1\rangle_{13}|\psi_{22}^2\rangle_{25}, \tag{A. 17}$$

$$|G_{41}\rangle = |\psi_{01}^1\rangle_{13}|\psi_{01}^2\rangle_{25} + |\psi_{23}^1\rangle_{13}|\psi_{23}^2\rangle_{25}, \tag{A. 18}$$

$$|G_{42}\rangle = |\psi_{01}^1\rangle_{13}|\psi_{02}^2\rangle_{25} + |\psi_{23}^1\rangle_{13}|\psi_{20}^2\rangle_{25}, \tag{A. 19}$$

$$|G_{43}\rangle = |\psi_{01}^1\rangle_{13}|\psi_{03}^2\rangle_{25} + |\psi_{23}^1\rangle_{13}|\psi_{21}^2\rangle_{25}, \tag{A. 20}$$

$$|G_{50}\rangle = |\psi_{11}^1\rangle_{13}|\psi_{10}^2\rangle_{25} + |\psi_{33}^1\rangle_{13}|\psi_{32}^2\rangle_{25}, \qquad (A.\,21)$$

$$|G_{51}\rangle = |\psi_{11}^1\rangle_{13}|\psi_{11}^2\rangle_{25} + |\psi_{33}^1\rangle_{13}|\psi_{33}^2\rangle_{25}, \qquad (A.\,22)$$

$$|G_{52}\rangle = |\psi_{11}^1\rangle_{13}|\psi_{12}^2\rangle_{25} + |\psi_{33}^1\rangle_{13}|\psi_{30}^2\rangle_{25}, \qquad (A.\,23)$$

$$|G_{53}\rangle = |\psi_{11}^1\rangle_{13}|\psi_{13}^2\rangle_{25} + |\psi_{33}^1\rangle_{13}|\psi_{31}^2\rangle_{25}, \qquad (A.\,24)$$

$$|G_{60}\rangle = |\psi_{21}^1\rangle_{13}|\psi_{20}^2\rangle_{25} + |\psi_{03}^1\rangle_{13}|\psi_{02}^2\rangle_{25}, \qquad (A.\,25)$$

$$|G_{61}\rangle = |\psi_{21}^1\rangle_{13}|\psi_{21}^2\rangle_{25} + |\psi_{03}^1\rangle_{13}|\psi_{03}^2\rangle_{25}, \qquad (A.\,26)$$

$$|G_{62}\rangle = |\psi_{21}^1\rangle_{13}|\psi_{22}^2\rangle_{25} + |\psi_{03}^1\rangle_{13}|\psi_{00}^2\rangle_{25}, \qquad (A.\,27)$$

$$|G_{63}\rangle = |\psi_{21}^1\rangle_{13}|\psi_{23}^2\rangle_{25} + |\psi_{03}^1\rangle_{13}|\psi_{01}^2\rangle_{25}, \qquad (A.\,28)$$

$$|G_{70}\rangle = |\psi_{31}^1\rangle_{13}|\psi_{30}^2\rangle_{25} + |\psi_{13}^1\rangle_{13}|\psi_{12}^2\rangle_{25}, \qquad (A.\,29)$$

$$|G_{71}\rangle = |\psi_{31}^1\rangle_{13}|\psi_{31}^2\rangle_{25} + |\psi_{13}^1\rangle_{13}|\psi_{13}^2\rangle_{25}, \qquad (A.\,30)$$

$$|G_{72}\rangle = |\psi_{31}^1\rangle_{13}|\psi_{32}^2\rangle_{25} + |\psi_{13}^1\rangle_{13}|\psi_{10}^2\rangle_{25}, \qquad (A.\,31)$$

$$|G_{73}\rangle = |\psi_{31}^1\rangle_{13}|\psi_{33}^2\rangle_{25} + |\psi_{13}^1\rangle_{13}|\psi_{11}^2\rangle_{25}. \qquad (A.\,32)$$

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